SAMPLE **SAMPLE**

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Topology Comprehensive Exam

Complete **SIX** of the following eight problems.

1. (a) State the Urysohn Lemma.
   (b) Show that a connected normal space with at least two points is uncountable.

2. Let A be a compact subset of a Hausdorff space X. Show that A is closed. (Your proof must be elementary and may not use nets.)

3. Suppose A and B are connected subsets of X. Suppose $A \cap B \neq \emptyset$. Show that $A \cup B$ is connected.

4. Let A be a subset of a topological space X. Suppose there exists a continuous function $r : X \to A$ such that $r(a) = a$ for all $a \in A$ (such a map is called a *retraction* onto A).
   (a) Show that r is a quotient map.
   (b) Show that A is closed.

5. (a) State the characteristic property of the product topology.
   (b) Let $\{X_\alpha\}_{\alpha \in A}$ be a family of topological spaces, and for each α let $W_\alpha \subseteq X_\alpha$. Let $X = \prod X_\alpha$ and let $W = \prod W_\alpha$. Without ever mentioning open or closed sets, prove that the product topology on W and the subspace topology on W (as a subspace of X) are the same.

6. (a) Define an n-manifold.
   (b) Show that $\{(x, y) \in \mathbb{R}^2 : x \neq 0, y = 1/x$ is a 1-manifold$\}$.

7. Let $B = \{x \in \mathbb{R}^2 : |x| \leq 1\}$, where $| \cdot |$ denotes the Euclidean norm. Define an equivalence relation on B by $x \sim y$ if $|x| = |y| = 1$. Prove that $B/\sim$ is homeomorphic to a familiar space.

8. Exhibit counterexamples (and brief justifications) to the following false claims.
   (a) Every open map is closed.
   (b) Connected components are open.
   (c) Every surjective continuous map is a quotient map.
   (d) If $A \subseteq X$ is path connected, so is $\overline{A}$.