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Culturally Based Math Education as a Way to Improve Alaska Native Students' Math Performance

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Abstract

Culturally based instruction has long been touted as a preferred approach to improving the performance of American Indian and Alaska Native (AI/AN) students' academic performance. However, there has been scant research to support this conjecture, particularly when quantitative data and quasi-experimental designs are included. The results of this study show that the culturally based math curriculum, *Building a Fish Rack: Investigations into Proof, Properties, Perimeter, and Area,* enabled sixth grade Yup'ik students and their urban counterparts to increase their mathematical understanding of perimeter and area.

The study involved one semester's worth of data (258 students in 15 classes). The study was a strong quasi-experimental design with random assignment and the results were based on pre- and post-test score differences. The study involved one urban school district, Fairbanks, and four rural school districts with approximately a 97% Yup'ik population. The study showed that the difference in test results between all treatment groups and all control groups was significant beyond the accepted standard of $p<0.05$. Although the urban treatment group gained the most from this curriculum, the most important finding is that the rural treatment group outperformed the rural control group at a significant level beyond the accepted standard of $p<0.05$. The study is encouraging, as it shows that the treatment effect on Yup'ik students narrows the long-standing academic gap when comparing that group's and the Yup'ik control group's relative performance against the urban control group. Further studies are necessary to determine if the results can be replicated, if the results are tied to a specific topic area, and if a study that uses complementary research methods can unpack the factors behind the gain.

Culturally Based Math Education as a Way to Improve Alaska Native Students' Math Performance

> In authentic cultural mediation, school practices are an extension of cultural knowledge associated with customs, traditions, and values from the students' home-culture. The curriculum content is based on knowledge valued by the local community and reflects the history and culture of its people (Hollins, 1996, p. 140).

Introduction

In many indigenous, third world, and rural contexts the school and its curriculum (what knowledge is included), teaching (ways of communicating, relating, and valuing), and evaluating often do not represent the local culture. Over the better part of the past century, research reports on American Indian and Alaska Native (AI/AN) students have called for educational programs that connect the culture of the community to the culture of the school, including the use of local languages, local knowledge, and local involvement (Meriam, Brown, Cloud, & Dale, 1928; Deyhle & Swisher, 1997; Swisher & Tippeconnic, 1999; and Pavel, 2001). These reports strongly suggest that the cultural divide (often referred to in the literature as the cultural mismatch theory) between school and community is a major factor causing the persistent gap between the academic performance of AI/AN students and their non-native peers. These factors, as well as rapid teacher turnover, teachers teaching out of their content area, and the low percentage of local teachers (Pavel, 1998) resulted in AI/AN students' underperformance in core academic subjects, particularly mathematics. (Appendix A shows differences in some selected math test scores for rural and urban Alaskan students).

Students bring with them experiences, cultural practices, and, in fact, intuitive mathematical knowledge (see S. Adams, 2003 for a fuller description). Cross-cultural cognitive psychologists and anthropologists have described out-of-school mathematical learning (Sternberg et al., 2001, Nokes, Geissler, Prince, Okatcha, Bundy, & Grigorenko; Lave, 1988).

These studies have described everyday mathematical practices from carpet laying (Massingila, 1994) to Brazilian street children selling candy and learning their own algorithms for making change (Nunes, Schliemann, & Carraher, 1993); to how numbers are organized, grouped, and how counting occurs by various cultural groups (Saxe, 1981; Denny, 1986; Lipka, 1994); to various systems of body measures (Lipka, Mohatt, and the Ciulistet Group, 1998).

Similarly, in our work in southwest Alaska, Yup'ik Eskimo elders have their own system of perceiving, dividing, and locating points in space; this is supported by a rich lexicon of spatial words and experiences situated in traveling across the vast (undifferentiated to outsiders) tundra; and they have their own system and way of quantifying and counting including a base 20 system with sub base five. The embedded mathematics is part of their cultural and linguistic heritage and it continues to be reinforced today by living off the bounty of land and sea.

Other educational researchers, linguists, and anthropologists have studied the process of learning in and out of school (how knowledge is transmitted and learned) across a variety of cultures. For example, the disjunction between communication processes in school and out of school within indigenous communities has shown how this leads to miscommunication and barriers to in-school learning (Au, 1980; Mohatt and Erickson, 1981; Phillips, 1983; and Brenner, 1988). The cultural mismatch theory supported by the above research makes a persuasive argument that excluding local knowledge and ways of communicating impedes performance of AI/AN students. Yet, two recent reviews of the literature on culturally based education (CBE) yielded very few CBE studies that used a quasi-experimental design and showed statistical significance (Lipka, 2002; Demmert, 2003); and in terms of mathematics even a smaller number were identified (Brenner, 1998; Lipka and Adams, 2001).

The major purpose of this intervention study was to determine if a culturally based math curriculum (treatment and independent variable) would increase sixth grade students' mathematical thinking and performance (dependent variable), particularly among Yup'ik students from southwest Alaska.

Background

Southwest Alaska today remains physically separated from the rest of the state; there are no roads that connect it to Anchorage or other regional towns or villages. Subsistence (hunting and gathering food) remains a vital part of people's lives. Whole villages continue to shift to summer camps to harvest salmon. Schools range in size from less than 50 to more than 500 students. Many schools have multi-grade level classes, and recently more schools and communities are supporting Yup'ik immersion language programs in response to the weakening of the Yup'ik language (Krauss, 1980; Tull, 1998).

The small percentage of local teachers in southwest Alaska, as in the rest of the state, spurred the establishment of the Cross-Cultural Education Development Program (X-CED) in the late 1960s. This teacher education program was a part of the University of Alaska Fairbanks. It arose at the end of the Johnson administration as an attempt to create social equity within education. Its purpose was to have local educators who would be committed to teaching in the small villages throughout the state, and to have a cadre of teachers who could bridge the cultural divide between school and community.

One of the authors[1] was a field-based teacher educator in southwest Alaska and he and local teachers, elders, and researchers began working together during the late 1980s to connect community knowledge and school knowledge. They received grants to adapt elders' knowledge into mathematical knowledge. This arduous process culminated in a series of supplemental

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[1] Jerry Lipka was a field faculty member in the Bristol Bay region of Alaska from 1981 until 1992. For a detailed description on this program and the work that laid the foundation for this study see Lipka, Mohatt, and the Ciulistet Group (1998) *Transforming the Culture of Schools: Yup'ik Eskimo Examples.*

elementary school mathematics modules. Topics ranged from pre-algebraic thinking to grouping and place values to perimeter and area.

The Curriculum

Each supplemental math module has a cultural storyline such as gathering eggs or berries. The mathematics of the module flows from the Yup'ik culture. For example, *Going to Egg Island: Adventures in Grouping and Place Values* is a module about grouping and place value using the Yup'ik system of counting, base 20 and sub base five. Some modules do not base the mathematics on the culture, but do use familiar local stories, games, and visuals to make the curriculum accessible to students. In keeping with the CBE model, the modules include expert-apprentice modeling and peer instruction adapted from experienced Yup'ik teachers and elders. The CBE component also includes a hands-on approach to mathematics, with an emphasis on spatial visualization and spatial manipulations. Further, traditional and contemporary stories are part of this integrative approach.

Mathematically the modules were developed to be challenging and problem-based, and to develop students as flexible thinkers. This is one of the themes that elders repeatedly talked about. Therefore, the modules did not teach standard algorithms, nor emphasize procedural knowledge such as memorizing formulas for perimeter and area. Instead they connected to experiences students may have already been familiar with, provided hands-on experiences through design work, and challenged students to make conjectures and provide evidence. Students were not viewed as consumers of mathematical manipulatives but instead as producers of mathematical knowledge, albeit appropriate for the grade and developmental level. For example, this meant that students would make their own graphs, label them, and grapple with issues of scale. They would take information contained in the graph and summarize it in a table.

The Mathematics of the Module

Results in this paper stem from research around one particular module: *Building a Fish Rack.* This module's central cultural and mathematical theme is the building of a fish rack (fish racks are structures used for drying salmon). The core mathematical concepts related to designing and building a fish rack are similar to any rectangular structure. The module was taught to students in a three- to six-week period.

The curriculum follows the way some Yup'ik elders construct a fish rack. It asks the students to find the corners of an approximately nine-by-twelve-foot rectangular base. Students cannot use standard measures. Students may use body measures; create their own unit of measure, or use small (less than five feet long) locally available materials. This activity unfolds into a series of explorations around what a rectangle is, how students know they have a rectangle (notions of conjecture and proof), other quadrilaterals and how they are related, as well as perimeter and area problems, particularly changes in dimensions and area when perimeter is held constant. Thus by connecting to a common Yup'ik activity based on the salmon summer fishing season, students were encouraged to learn about physical proofs of the properties of a rectangle as they attempted to solve a practical problem—how to determine they have a rectangular base—related to building a structure.

Hypotheses of the Study

The hypotheses of this study flow from the brief review of the literature and include the following. (1) Yup'ik treatment students (those who learn the concepts of perimeter and area from the *Building a Fish Rack* module) from villages in southwest Alaska will outperform Yup'ik control students also from villages in southwest Alaska. (2) Students (urban and rural) in the treatment group will outperform the control group students on overall mathematical performance. (3) Students in the treatment group will understand the dynamic relationship of properties associated with a rectangle, such as perimeter, dimensions, and area better than those in the control group. That is, they will outperform the control group on questions that measure students' knowledge relating to the functional relationship of perimeter and area to dimensions when perimeter is held constant. (4) Treatment students will outperform control group students on the concept of area applied to various shapes such as triangles, rectangles, parallelograms, trapezoids, and circles.

Research Questions

Before we concern ourselves with the technical aspects of comparability between groups (urban and rural and treatment and control) it is important to reiterate that our research questions flow from the long standing body of literature that documents the wide discrepancy in performance between AI/AN and their nonnative contemporaries. To determine if the rural Yup'ik treatment group outperforms the rural Yup'ik control group and if the treatment has an overall effect, it is important to satisfy the following set of questions.

The focus of our research concerned the population of students and their comparability. We needed to establish whether different groups of students had the same starting point, as measured by a pre-test. Our specific groups for comparison were treatment and control, as well as rural and urban. Once we determined the comparability of the treatment and control groups, we wanted to know if the culturally based math curriculum enhanced students' mathematical learning and performance. Similarly, we were interested in finding out if the treatment effects (if any) favored any group, that is the rural predominantly Yup'ik population versus the mostly Caucasian urban population. We were also interested in determining the efficacy of the curriculum; what aspects (if any) showed improved mathematical performance—definitional knowledge of shape (including proof of a rectangle), perimeter (solving one-step problems and calculating), area (solving one-step problems and calculating), and the dynamic relationship between perimeter (when held constant), dimensions, and area. Lastly, we were interested in learning about absolute and relative scores on the sub-items indicated above and what those scores tell us us about the module and students' knowledge of perimeter and area.

Research Design

To determine the treatment effects of the curriculum *Building a Fish Rack*, we designed a two-by-two matrix with columns representing the treatment group (culturally based math) and the control group (textbook) and rows representing urban and rural. Table 1 shows the research design for spring 2001. The curriculum module under study is designed to be taught over a three- to six-week period. The module, as well as the control curriculum, were administered to their respective groups over the course of three weeks in Spring 2001.

Table 1: Experimental Design for Spring 2001

	Treatment	**Control**	**Total**
Urban (Fairbanks)	5 classes 109 students	3 classes 71 students	8 classes 180 students
Rural (Yup'ik)	4 classes 51 students	3 classes 27 students	7 classes 78 students
Total	9 classes 160 students	6 classes 98 students	15 classes 258 students

Teacher volunteers were solicited from the following school districts: Southwest Region Schools, Lower Yukon School District, Yupiit School District, St. Mary's School District, and

Fairbanks North Star Borough School District.

Random Assignment of Teachers and Teacher Training Workshop

We invited and held a workshop for the teacher volunteers, who were then randomly assigned to the treatment or control group. We offered a workshop for both treatment and control teachers as one way to mitigate the Hawthorne effect, providing an overview of the project and reviewing the concepts of perimeter and areas as they are addressed in a commonly used textbook. It was our belief that all teachers (treatment or control) thought that they were getting something special. However, we could not hide the fact that some teachers used the treatment module while others did not. In addition, we reviewed all procedures for this study—the pre- and post-tests, classroom observations, interviews, and videotape sessions—and scheduled an audio conference to discuss the module. The workshop included a discussion of the pedagogical portion of the treatment—a constructivist approach in a cultural context—and specific instructions about how to teach the module.

Two teachers from Fairbanks who previously taught this module were assigned to the treatment group to avoid confounding results.

Data Collection and Analysis Methods

All students were given a pre-test on their mathematical knowledge concerning shape, perimeter and area immediately before the teachers began teaching perimeter and area. Immediately upon completion of the unit on perimeter and area, all students took a post-test. (See Appendix B and Appendix C for the pre- and post-tests).

Students' pre-and post-tests were mailed to us immediately after the teachers completed teaching the modules. The tests were mailed to us. Teachers also sent us student work, including journals and classroom artifacts; some teachers also sent video and audiotapes of their classrooms. The analysis of videotapes of classroom interactions was beyond the scope of the present study.

We used a strong quasi-experimental research design, which is similar to an experimental design in all aspects except that we randomized teachers (not students) and then tested for equal starting points for students by group (treatment and control). Gliner and Morgan (2000) indicate the pre-test/post-test control group design is the one most commonly used in randomized experimental designs; therefore, we chose this approach. We defined our outcome measure as the gain differences between scores on pre- and post-tests per student. Other analytic methods that use post-tests only do not account for differences in starting points or for change over time (Gliner & Morgan, 2000). Depending on the starting points of each group, either a standard t-test or an analysis of variance (ANOVA) multiple regression technique was performed. To ensure equal starting points, we also used a t-test to consider comparability of pre-test scores per block and a comparison of individual test items between treatment and control groups.

We visited approximately six classrooms, observed students participating in the activities, interviewed students and teachers on their reactions to the activities, and videotaped the lessons. Participating teachers filled out a small exit survey form and participated in an audio conference. The videotapes will be analyzed at a later date to assess the differences between the treatment and control groups regarding communication among the students, student interest in learning, specific student thinking, and the ability of students to grasp the concepts taught them.

Developing the Instrument: Pre- and Post-Test Items on Perimeter and Area

Using questions from the National Assessment of Educational Progress (NAEP) and the Third International Mathematics and Science Study (TIMSS), along with self-generated additional items reflecting the mathematics of the module, we developed test items on the concepts of shape, perimeter, and area. The pre- and post-tests were piloted in the fall of 2000 and revised based on the piloting.[2]

[2] A Pilot study was conducted during the fall of 2000 entitled *Triarchic Intelligence, Culturally Based Mathematics, and School Improvement* in collaboration with Sternberg, Grigorenko, Newman, and Wildfeuer.

The pre-test had 14 questions, each worth five points, and three questions worth 10 points. The post-test had 13 questions, each worth five points, two questions worth 10 points, and two questions worth 7.5 points each. Partial credit was given in two cases, when there were multiple correct responses for multiple-choice problems and when students demonstrated partial knowledge to open-ended questions. Thus the raw score is equivalent to a percentage score and is presented in the latter manner throughout the paper. Because of limitations of sample size we were not able to include a post-test only control group to determine the effect of pre-test on post-test. Therefore, we intentionally made the post-test slightly more difficult than the pre-test. Our reasoning followed that used by the "Solomon Four-Group Design" (Gliner & Morgan, 2000); its purpose is to mitigate the effects of the pre-test on the post-test. (See Appendix D for a more complete description of the development and comparability of the pre- and post-test items).

Analysis and Findings

Pre-test Data

Before we could analyze gain differences between treatment and control groups, we needed to ascertain if the treatment and control populations had similar starting points, as measured by their scores on the pre-test. Results of a two-tailed t-test show that there was no statistically significant difference between the pre-test scores for the treatment and control groups during spring 2001; the p-value[3] is high at 0.13, as shown in Table 2 below. We note that the t-test assumptions, which include independent samples drawn from normal populations with equal variances, were satisfied in this case. (An F-test for equal variances was conducted

[3] Statistically, the p-value is the smallest measure of a Type I error at which you can reject a hypothesis. Saying that the p-value is 0.13 is equivalent to stating a confidence level of 87%. Typically, if the p-value is smaller than 0.05, relating to a confidence level of 95% or more, then we have statistical significance.

showing the assumption of equal variances to hold with a significant p-value beyond the accepted standard of 0.01, in fact, p = 0.41). In summary for spring 2001, the treatment group's starting point was approximately 37% and the control group's was approximately 40%, which statistically are not considered to be different.

Table 2: Results of t-test for comparison between pre-test scores of treatment and control groups

	Treatment	**Control**
Mean	36.62	39.65
Variance	250.58	239.17
Observations	160	98
Pooled Variance	246.26	
Hypothesized Mean Difference	0	
Degrees of Freedom	256	
t Stat	-1.504	
P(T<=t) one-tail	0.067	
t Critical one-tail	1.65	
P(T<=t) two-tail	0.134	
t Critical two-tail	1.97	

We were also interested in the question: Do the rural and urban students have the same starting point? We performed a similar analysis using all pre-test scores for the urban and rural groups regardless of treatment or control. Since we expected the urban students to start at a higher level, we used a one-tail t-test. Additionally, based on confirmation of equal variance, an equal-variance t-test based was performed. The results show that the average starting levels of 41% for the urban group and 31% for the rural group were statistically significantly different beyond the accepted level of 0.01 (in fact, p~0). Thus at the 99% level, we can say that the urban students were starting at a higher level of knowledge in area and perimeter than the rural students. See Table 3 below.

Table 3: Results of t-test comparing pre-test scores of urban and rural groups

	Urban	Rural
Mean	40.71	30.99
Variance	222.31	242.30
Observations	180	78
Pooled Variance	228.32	
Hypothesized Mean Difference	0	
Degrees of Freedom	256	
T Stat	4.75	
P(T<=t) one-tail	0.000002	
T Critical one-tail	1.65	
P(T<=t) two-tail	0.000003	
T Critical two-tail	1.97	

Treatment Effects

The main research question was, “Does the treatment, the culturally based math curriculum, enhance students’ mathematical learning and performance?” Based on the results of the pre-test analysis discussed above, the gain difference in pre- and post-test scores were compared within blocks to remove the confounding effects of the urban group starting at a significantly higher level than the rural group.

Table 4 shows that the gain between the pre- and post-test scores for the urban treatment and urban control groups is approximately 16% and 4%, a difference of 12 percentage points. The difference in gain score between the two groups was statistically significant beyond the accepted standard of $p < 0.01$ (in fact, $p \sim 0$). Therefore, subject to the assumptions, the treatment did improve students’ math performance concerning the concepts of properties of a rectangle, perimeter, and area.

Table 4: Results of t-test comparing gain scores of urban treatment and control groups

	Urban Treatment	**Urban Control**
Mean	16.41	3.79
Variance	251.71	242.40
Observations	109	71
Pooled Variance	248.05	
Hypothesized Mean Difference	0	
Degrees of Freedom	178	
t Stat	5.25	
P(T<=t) one-tail	0.0000002	
t Critical one-tail	1.65	
P(T<=t) two-tail	0.0000004	
t Critical two-tail	1.97	

Table 5 shows that the gain between the pre- and post-test scores for the rural treatment and rural control groups is approximately 7% and –1%, a difference of 8 percentage points. The difference in gain score between the two groups was statistically significant beyond the accepted standard of $p < 0.02$ (in fact, $p = 0.015$). Therefore, the treatment did, subject to the assumptions, improve students' math performance in the properties of a rectangle, perimeter and area for the rural students as well.

Table 5: Results of t-test comparing gain scores of rural treatment and control groups

	Rural Treatment	**Rural Control**
Mean	6.98	-1.35
Variance	187.52	216.59
Observations	51	27
Pooled Variance	197.46	
Hypothesized Mean Difference		
Degrees of Freedom	76	
t Stat	2.49	
P(T<=t) one-tail	0.007	
t Critical one-tail	1.67	
P(T<=t) two-tail	0.015	
t Critical two-tail	1.99	

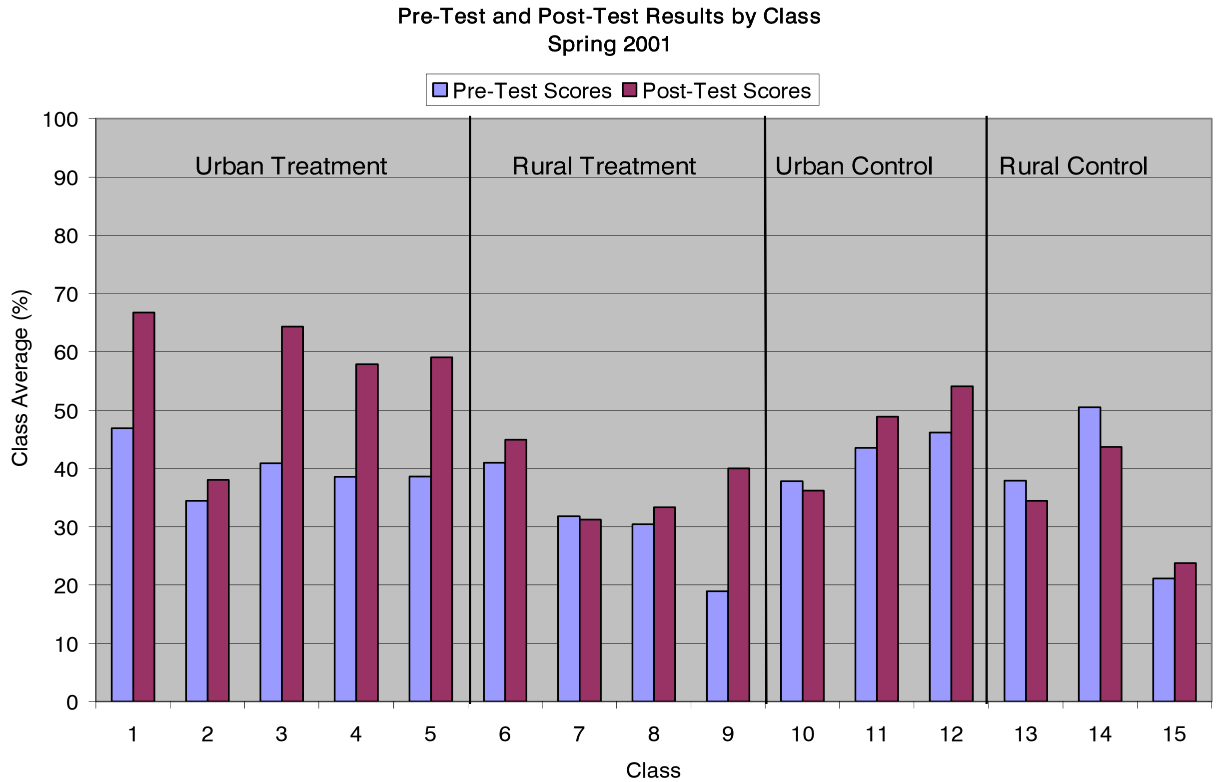
We were interested in which group received the largest gain from the treatment—rural or urban. In fact, the treatment helped the urban students most of all, as is evident with the 16%

gain for urban students (reported in Table 4) versus the 7% gain for rural students (reported in Table 5). Table 6 shows the results of the one-tail t-test for this comparison. Thus, the urban treatment group outperformed the rural treatment group at a statistically significant level beyond the accepted $p < 0.01$ (in fact, $p \sim 0$).

<u>Table 6: Results of t-test comparing gain scores of urban and rural treatment groups</u>

	Urban Treatment	**Rural Treatment**
Mean	16.41	6.98
Variance	251.71	187.52
Observations	109	51
Pooled Variance	231.40	
Hypothesized Mean Difference		
Degrees of Freedom	158	
T Stat	3.65	
P(T<=t) one-tail	0.0002	
T Critical one-tail	1.65	
P(T<=t) two-tail	0.0004	
T Critical two-tail	1.98	

Figure 1 summarizes and shows an overview of the average pre-test and post-test results by classroom, organized by blocks: urban treatment, rural treatment, urban control, and rural control groups. Further analysis of the data shows that when comparing classrooms within the same school, the treatment group outperformed the control group. For example, on Figure 1, rural treatment group No. 9 and rural control group No. 15 were classes in the same school, with pre-test scores of 19% and 21% respectively; on the post-test, however, scores of the treatment group improved 21 points, to an average of 40%, whereas the control group improved only 3 points, to an average of 24%. In practical terms, this difference means that the treatment group on average answered four more questions correctly than the control group. Therefore, the results show both a statistical significance and a practical significance.



<u>Figure 1: Comparison of average pre- and post-test results by class</u>

Figure 2 shows the average pre- and post-test results by block in comparison to each other. Interestingly, the rural treatment group's average post-test score was approximately 37%, while the urban control pre-test score was approximately 42%. We know from the statistical test performed in this study that the rural groups started out at a significantly lower point than the urban groups. By the end of the study, however, the rural treatment group narrowed the academic gap when compared with the urban control group. The gap between the rural treatment group and urban control group at the pre-test was 12%. At the end of the study, the gap had narrowed to 9%. Conversely, the difference between the rural and urban control group pre-test averages was 10% and for the post-test the difference was 15%.

We can surmise from this that the rural treatment group narrowed the existing academic gap. Yet the rural treatment group fell further behind the urban treatment group. The rural treatment group started out with a 9% difference from the urban treatment group on the pre-test; the difference grew to 19% on the post-test. Yet this must be viewed in light of the fact that the urban treatment group gained substantially more than any other group. Our assumption is that the control groups represent the present school conditions and that comparing the rural treatment group to the control groups is a valid indicator of the treatment effect. We also recognize that it could be argued that the treatment widened the gap between urban and rural groups. We will continue to be mindful of these varying interpretations in follow-up studies.

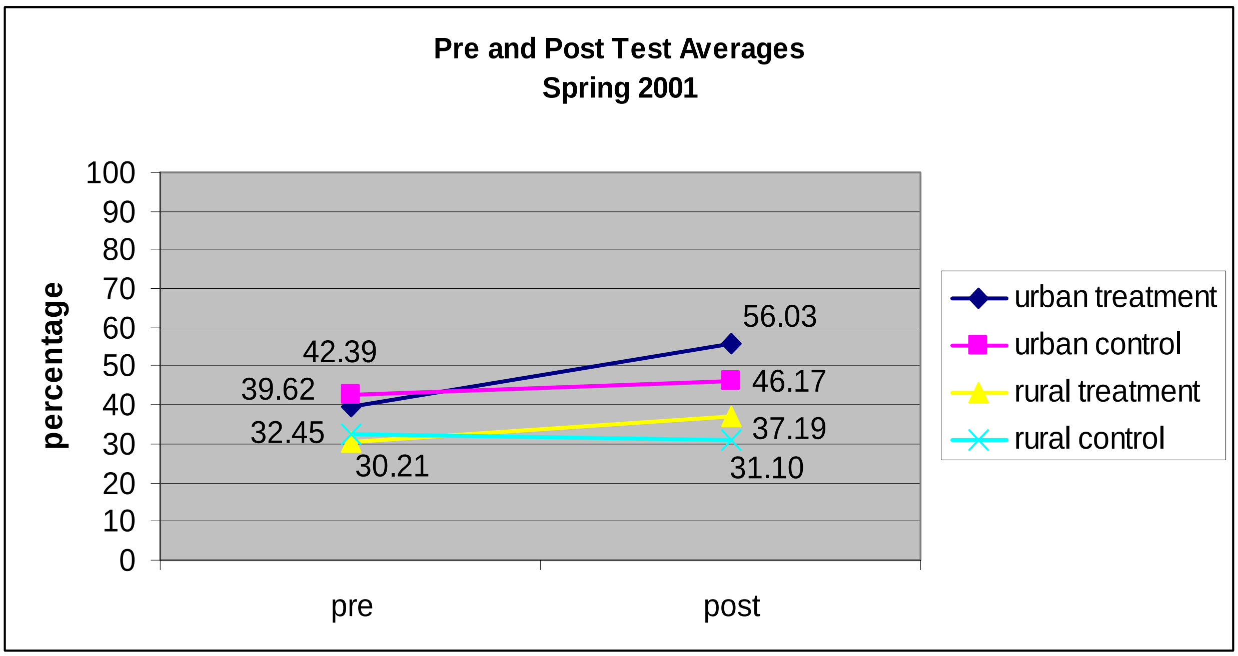


Figure 2: Comparison of pre- and post-test averages by block

Lastly, we were interested in the impact that the curriculum had on the treatment groups when compared with the control groups, specifically as it concerned post-test items No. 8 and No.10c. The former question involves the students describing or drawing (or both) a "proof" for a rectangle; the latter question concerns the dynamic relationship between perimeter and area when perimeter is held constant. These items, in particular, address important aspects of the module.

The results are shown in Figure 3 below. The average score for the urban treatment group was 51%, compared with approximately 10% for the urban control group. The rural treatment group average was 23%, compared with 8% for the rural control students. This is especially rewarding as it shows a treatment effect in which the rural treatment group outperformed the urban control group.

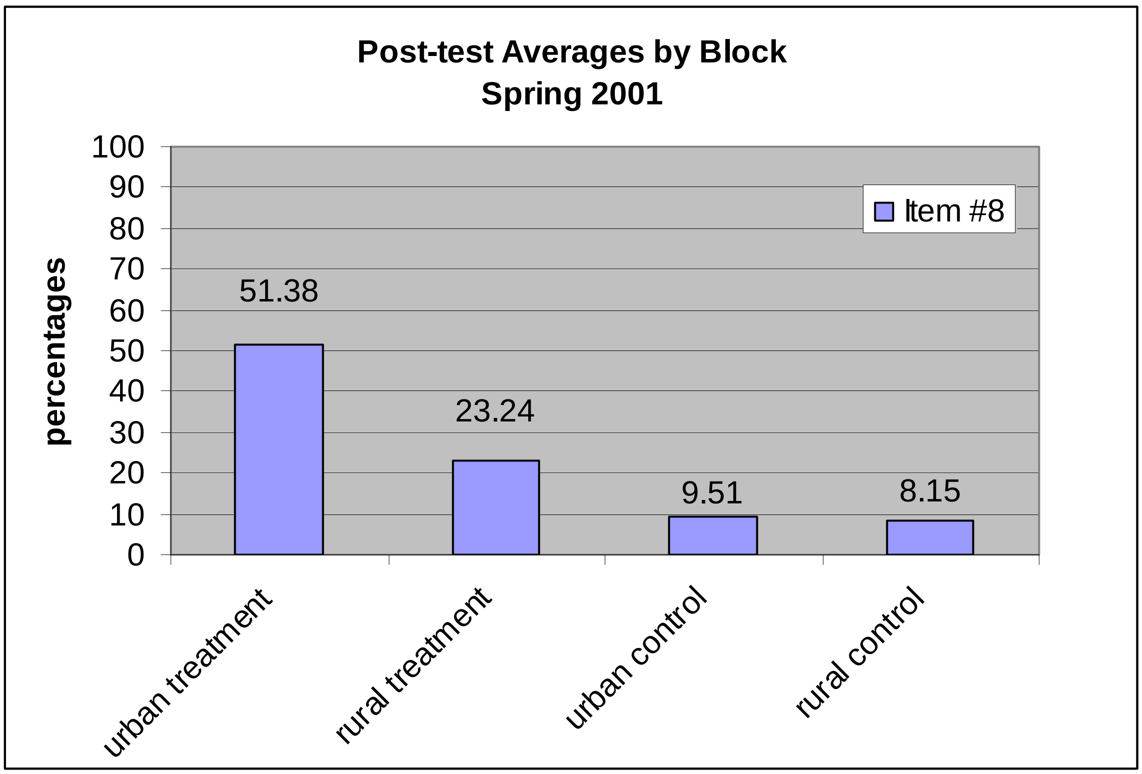


Figure 3: Comparison of post-test averages for Item No. 8 by block

Table 7 shows the results of the one-tail t-test (assuming unequal variances) analysis showing the difference in results between the treatment and control groups was statistically significant beyond the accepted level of $p < 0.01$ (in fact, $p \sim 0$).

Table 7: Results of t-test comparing scores on item No. 8 from treatment and control groups

	Treatment	Control
Mean	0.42	0.09
Variance	0.198	0.054
Observations	160	98
Hypothesized Mean Difference	0	
Degrees of Freedom	251	
t Stat	7.87	
P(T<=t) one-tail	0.00000000000005	
t Critical one-tail	1.65	
P(T<=t) two-tail	0.0000000000001	
t Critical two-tail	1.97	

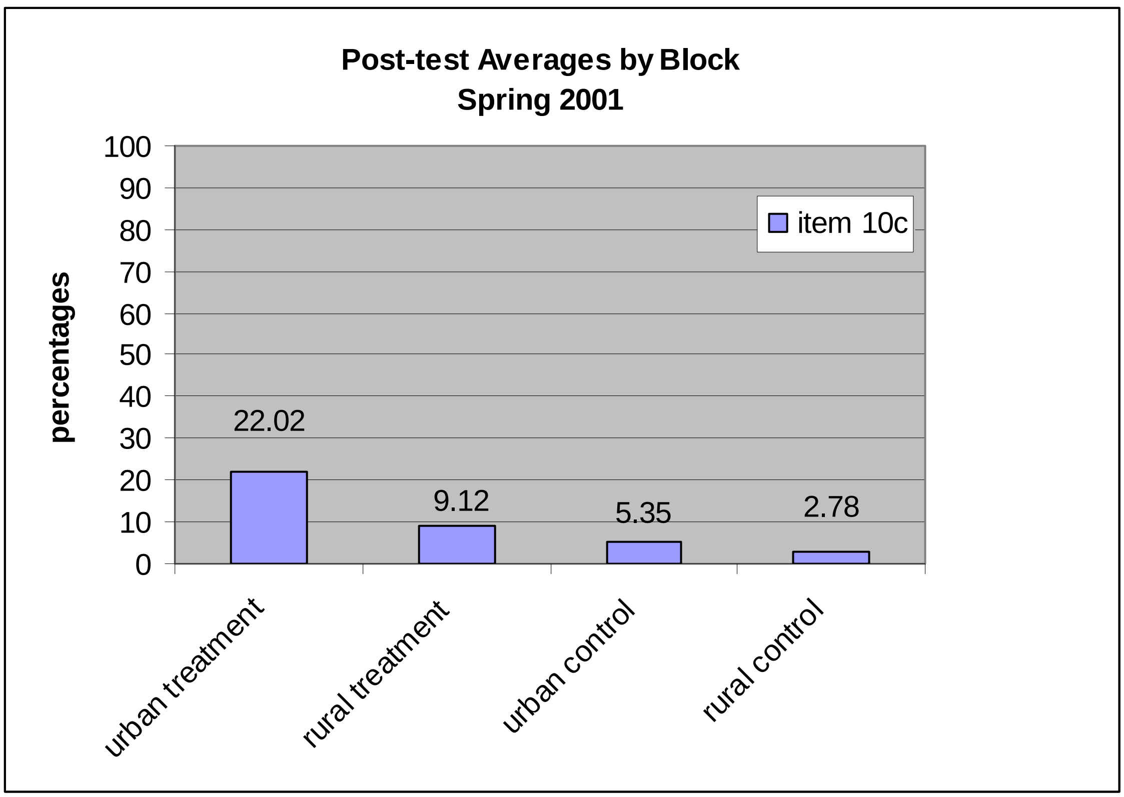


Figure 4: Comparison of post-test averages for Item No. 10c by block

Figure 4 shows that the treatment group outperformed the control group on item #10c (perimeter held constant and area changes) from the post-test. Again we see that the average scores of 22% and 10% for the urban and rural treatment groups, respectively, is substantially higher than the control groups scores of 5% and 3% for the urban and rural groups. Again, the rural treatment group outperformed the urban control group.Figure 4: Comparison of post-test averages for Item No. 10c by block

Table 8 shows that when comparing the treatment group (not differentiated by urban and rural) versus the control group on this item the results are statistically significant beyond the accepted level $p < 0.01$ (in fact, $p \sim 0$).

Table 8: Results of t-test comparing scores on item No.10c from treatment and control groups

	Treatment	**Control**
Mean	0.18	0.05
Variance	0.059	0.019
Observations	160	98
Hypothesized Mean Difference	0	
Degrees of Freedom	254	
t Stat	5.62	
P(T<=t) one-tail	0.00000002	
t Critical one-tail	1.65	
P(T<=t) two-tail	0.00000004	
t Critical two-tail	1.97	

Discussion

While the results of this study were in fact statistically significant for the treatment, aculturally based curriculum, the interpretation of these results must be viewed cautiously. This study does show a high probability in favor of the treatment effect, at the p-value level of 0.01. However, it was beyond the scope of the present study to do a full-scale qualitative and quantitative study in which the treatment and control classes could be monitored and observed periodically, and teachers and students could be interviewed. Having only partial knowledge of how the treatment and control classes were actually conducted, it is not known if all treatment teachers followed the module (fidelity of treatment) and if all control teachers followed the textbook. We do know that some treatment teachers, for example Teacher 2, did not teach the more challenging concepts of the module to her students, because she felt they were too difficult. In that case we see a diminished treatment effect as seen in Figure 1 when her scores are compared with the scores of other urban treatment teachers. Unfortunately, we do not have feedback from the rural treatment teachers to provide a further comparison. Moreover, since certain groups started at lower levels, it was naturally easier for these groups to show improvement. Yet there appears to be a positive treatment effect on the module's core concepts—physical proof of a rectangle applied to written argument, and illustration and perimeter held constant with area changing. In fact, most encouraging are the results gained from item No. 8 on the post-test, which asks students to describe or illustrate a proof for a rectangle. Although the evidence is strong for the treatment through comparisons of differences between pre- and post-test scores, there is also evidence of considerable weakness in students' understanding of perimeter and area. Detailed descriptions of student performance by item, in which items are color-coded per type, are shown in Figures 5 and 6. Blue represents area applied to squares and rectangles. Green represents area applied to more complex shapes such as parallelograms and circles. Pink represents perimeter, while red is used for the more dynamic aspects of perimeter. Yellow represents shape and proofs of a rectangle.

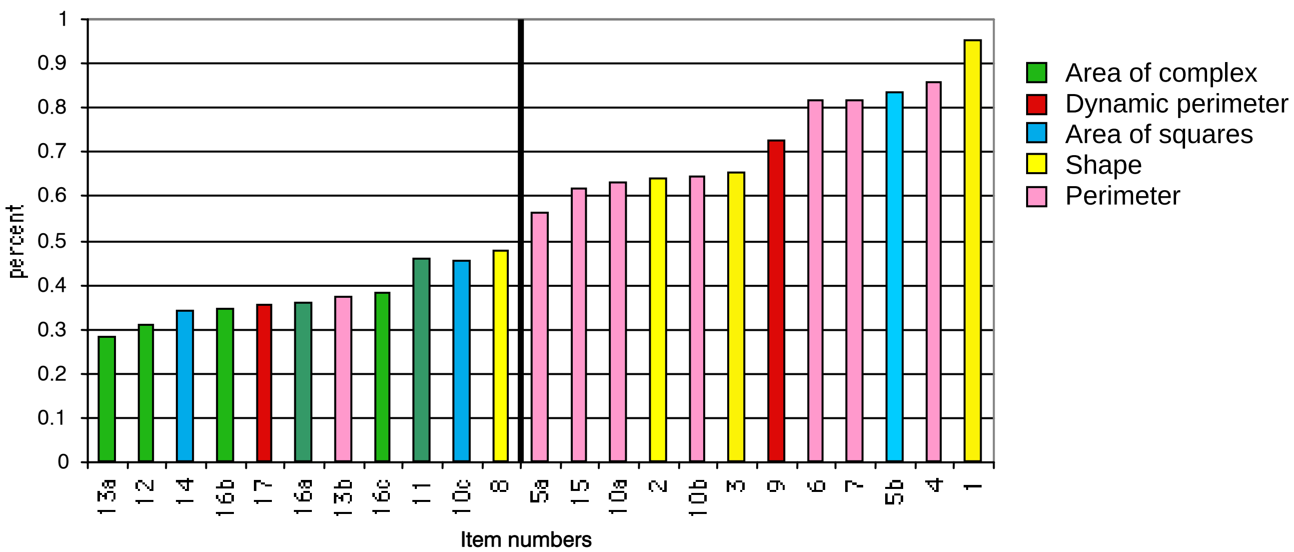


Figure 5: Post-test Item Analysis – Scores in Ascending Order for the Urban Treatment Group

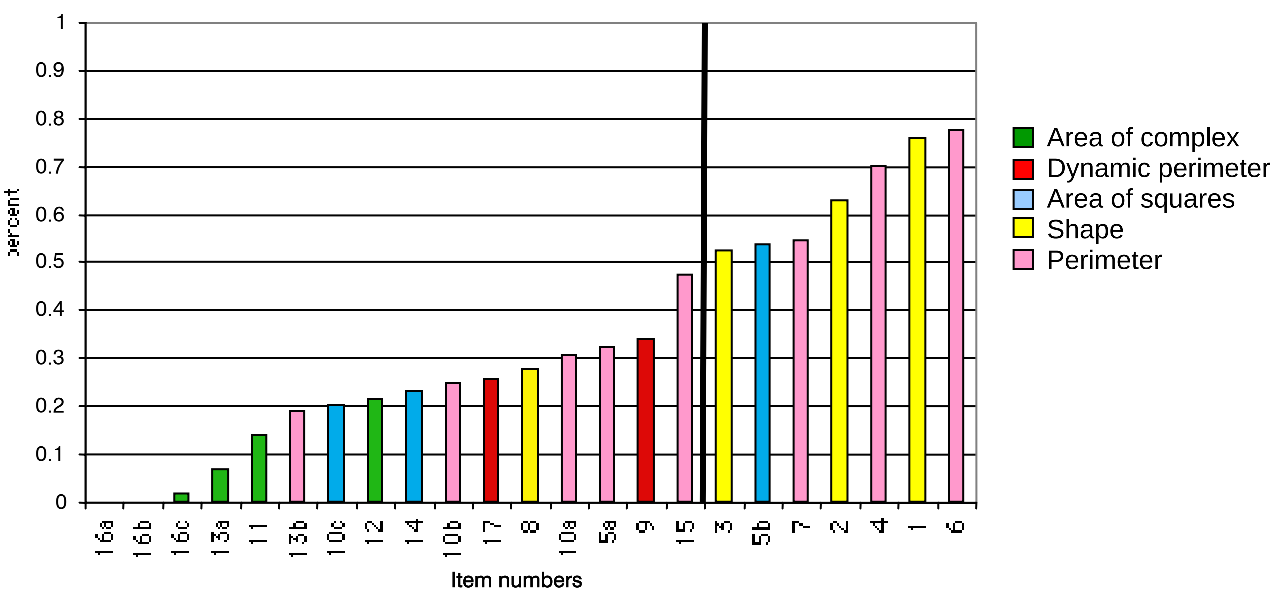


Figure 6: Post-test Item Analysis – Scores in Ascending Order for the Rural Treatment Group

These figures (5 and 6) show students had difficulty with the dynamic aspect of perimeter held constant, finding the area of parallelograms and trapezoids, and did far better on simple applications of perimeter and area. While reviewing and analyzing student work, we often saw students mistake the concept of perimeter for area and vice versa—for example, when illustrating a perimeter of 24 inches. The results of this study show some simple ways in which the curriculum itself could be modified and improved. For example, each activity needs to include the basic mathematical terms related to that activity. Nearly 23% of the urban control group and 25% of the rural treatment group did not know the difference between a parallelogram and other shapes. Also, students had difficulty with the word quadrilateral. Practice and use of these terms within the module would probably assist students. Aspects of the pre- and post-tests were not adequately aligned with the treatment; in fact, it was the treatment that needed revision. For example, although area was investigated within the module, students did not have an opportunity to practice or apply this knowledge. Hence, the curriculum has been revised to include more paper and pencil practice for the students. Although we included a one-day workshop for the teachers, the length and intensity of the workshop was not adequate to cover pedagogical, cultural, and mathematical aspects of the curriculum. In the future, we hope to make the appropriate changes.

There are reasons to believe that the study has underrepresented the effects of the treatment or the power of the curriculum. For example, teachers using the module should really be thought of as first-time users of a novel curriculum when compared with the control teachers, who more likely than not are well-practiced with their textbooks. Further, other treatment effects, such as the role of culture and how it affected student performance, were beyond the scope of the present study, although anecdotal data and teacher reports indicated that the module positively affected urban Native students. It is not possible to unpack the data further without further classroom observations. In addition, it was similarly beyond the scope of the project to measure the aspects of culture, geography, biology or other content knowledge areas that the students gained by using this module. We know that some of the rural students identified with the cultural story line, with ways of using body measures, and with the use of practical and everyday problems. In the future, we hope to continue this work and to address these issues.There are also limitations to this study. The first limitation deals with random sampling of teachers. Although we randomly assigned teachers to the treatment and control groups, all teachers involved in the project were volunteers. Also, two teachers had used the curriculum in the past and were assigned to the treatment group automatically to avoid confounding results. The next limitation worth noting concerns evaluation of the project. Although a third party evaluator was employed, she was involved mostly in classroom observations and so was not explicitly an outside evaluator. The final limitation focuses on the development of the pre- and post-tests. Although we attempted to create a fair test, there is always the possibility of bias favoring the treatment group over the control group.

Conclusion

These results are promising since the long-term gap between math performance for American Indian/Alaska Native students and Alaska's rural students has been, and continues to be, a significant and disturbing problem. The results of this study show that the *Building a Fish Rack* module, a culturally based, inquiry-oriented math curriculum, can improve performance differences in mathematics for rural Alaska Native (Yup'ik Eskimo) students. Further, the analysis of the data also shows the effects of this culturally based curriculum beyond Yup'ik students. This may mean that the unfolding of the curriculum occurs differently in cultural and geographical settings. Most importantly, this curriculum and research project begins to address the call for curricular and instructional approaches that are culturally and linguistically connected to AI/AN communities. Although we cannot conclude that the cause of the predominantly Yup'ik rural treatment groups' gain score was a function of culture, we can conclude that the rural, mostly Yup'ik, control group fell further behind their urban counterparts while the predominantly Yup'ik treatment group gained ground when compared with the urban control group. These promising first steps will be followed by additional research to replicate and expand the above study, as well as to investigate classroom factors that may contribute to student gain scores in more depth.

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APPENDIX A

CAT SCORES				
Area	***Grade***	***Top quart***	***Bottom quart***	***Percentile rank***
URBAN	4	46.20	13.10	71.75
RURAL	4	9.47	54.45	25.67

HSGQE RESULTS					
Area	***Grade***	***No. pass***	***% pass***	***No. Not Pass***	***% No. Pass***
URBAN	10	274	36.17	491.00	63.83
RURAL	10	8	7.41	112.00	92.59

BENCHMARK RESULTS			
Grade		***Urban***	***Rural***
3	**ADV.**	36.64	3.87
	PROF.	40.82	20.12
	BELOW	17.78	44.25
	NOT	4.82	31.78
6	**ADV.**	26.24	3.91
	PROF.	41.30	17.85
	BELOW	15.94	17.88
	NOT	16.54	60.36
8	**ADV.**	5.65	0.00
	PROF.	35.10	14.52
	BELOW	45.50	37.29
	NOT	13.75	48.20

(Data obtained from http://www.eed.state.ak.us/tls/assessment/results.html)

APPENDIX B

PRE-TEST

For

Building a Fish Rack Module

1. Circle the square.

2. Circle two properties of a rectangle.

   a. All angles are 90 degrees

   b. Opposite sides intersect

   c. Opposite sides are perpendicular

   d. Opposite sides are parallel

3. A quadrilateral is a (circle the correct response)

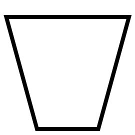
   a. A three-sided closed figure

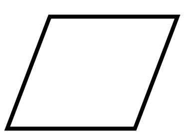
   b. A closed figure with three angles

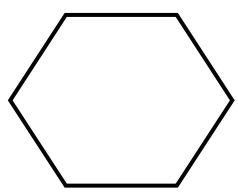
   c. A four-sided closed figure

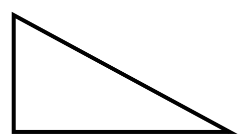
   d. Any shape with more than two sides

4. Circle the shapes that are quadrilaterals**.**

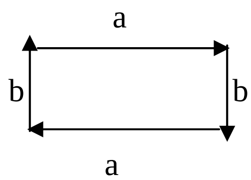
a. 

b. 

c. 

d. 

5. You want to put a border around the table for your sister’s birthday party. Circle the method you would use to measure the border.



(a) a + b + a + b **(b)** a × b **(c)** 2 (a) x 2 (b) **(d)** $\frac{2a + 2b}{2}$

6. You want to measure how much space something covers. What would you be measuring? Circle the correct answer.

a. area

b. perimeter

c. length

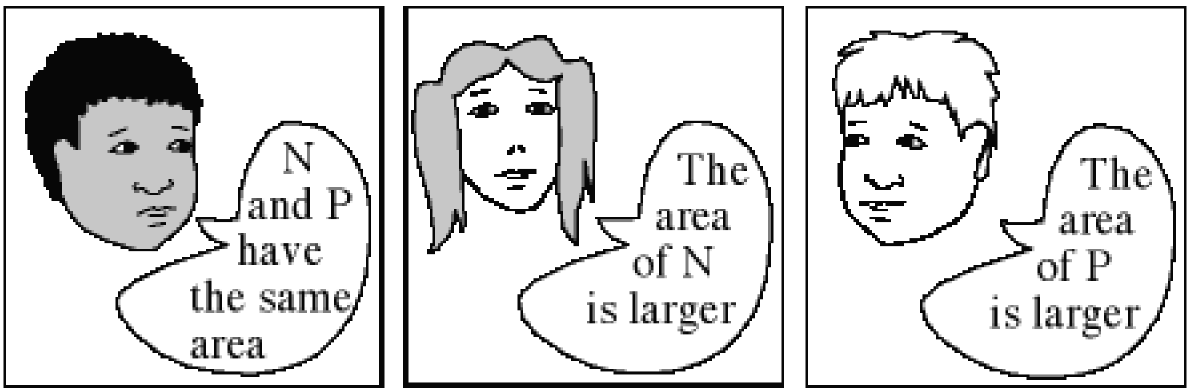
d. strength

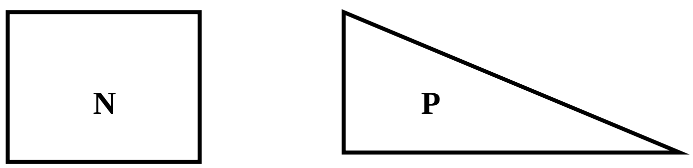
7. Draw two different shaped rectangles that have a perimeter of 16. Use the graph paper below and label the length and width for each rectangle.

D	R	A	W		R	E	C	T	A	N	G	L	E	#	1				
D	R	A	W		R	E	C	T	A	N	G	L	E	#	2				

8. The perimeter of a square is 20 inches. What is the length of a side? _____________

9. Bob, Carmen, and Tyler were comparing the areas of N and P.





Bob, Carmen, and Tyler were comparing the areas of N and P. Bob said that N and P have the same area. Carmen said that the area of N is larger. Tyler said that the area of P is larger. Students may use rulers, paper, and scissors for this problem.

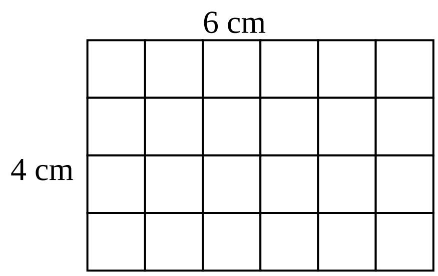
Who was correct? ________________________

Use words and pictures (or both) to explain why:

____

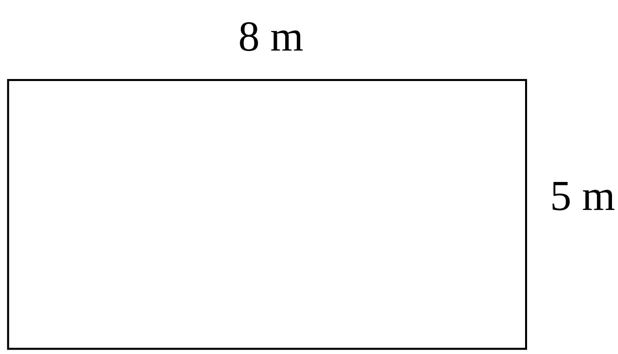
____

10. What is the area of this rectangle?



A) 4 square cm
B) 6 square cm
C) 10 square cm
D) 20 square cm
E) 24 square cm
F) I don't know

11. What is the PERIMETER of this rectangle?



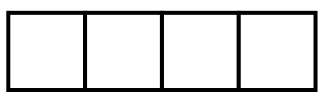
A) 13 meters
B) 26 meters
C) 40 meters
D) 80 meters
E) I don't know

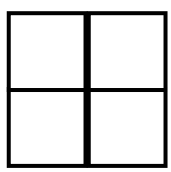
12. A. Draw a figure whose area is 30 square units.

B. Make up a word problem in which you need to find the area of an enclosed space.

C. Can two different figures have the same perimeter but different areas? Draw rectangles and explain your answer.

13. Here are two figures. Each side is 2cm. Circle all the correct answers.

1. 

2. 

A.

a. The perimeter of figure #1 and figure #2 are equal.
b. The perimeter of figure #1 is smaller than figure #2
c. The perimeter of figure #1 is larger than the perimeter of figure #2.
d. The area of figure #1 equals the area of figure #2.
e. The area of figure #1 is greater than the area of figure #2.

B. Calculate the perimeter of figures 1 and 2.

The perimeter of figure #1 = _____

The perimeter of figure #2 = _______

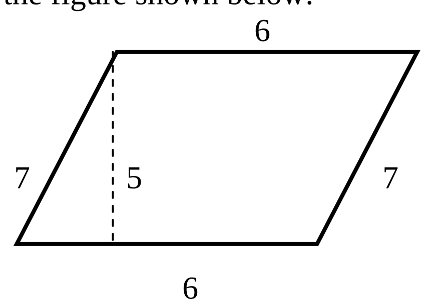
14. The formula for the area of a parallelogram is? Circle the correct answer.

a. A = 2L + 2W
b. A = 2L x 2W
c. A= B x H
d. A = $\frac{B \times H}{L + W}$

15. The circumference of a circle is the same as? Circle the correct answer.

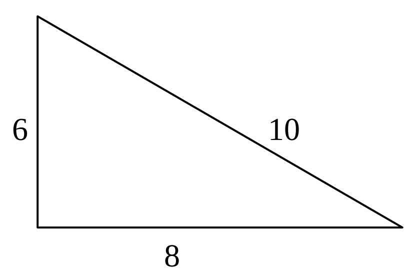
a. its area
b. perimeter
c. diameter
d. radius

16. Find the perimeter and area for the figure shown below:



a. Perimeter = ___________
b. Area = ____________

17. Find the perimeter and area of the following triangle.



a. perimeter equals = _______
b. area equals = ___________

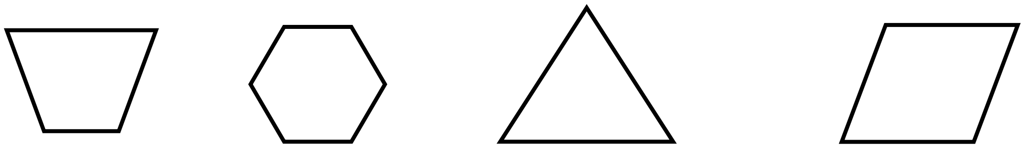
APPENDIX C

POST-TEST

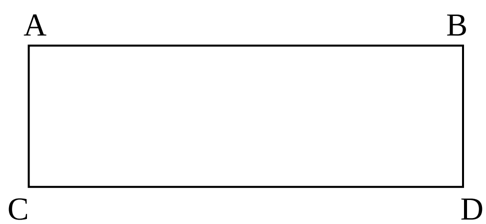
For

Building a Fish Rack Module

1. Circle the parallelogram.



2. Here is a rectangle. Circle below the statements which accurately describe this rectangle.



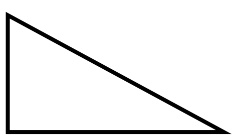
a. $\overline{AC}$ is parallel to $\overline{AB}$

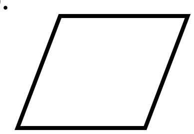
b. $\overline{AC}$ is parallel to $\overline{BD}$

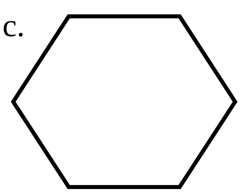
c. $\overline{AB}$ is parallel to $\overline{CD}$

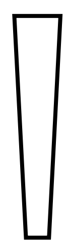
d. $\overline{AB}$ is parallel to $\overline{BD}$

3. Circle the shape(s) that is or are not quadrilaterals.

a. 

b. 

c. 

d. 

4. You walked around the outside of a small lake and want to know how far you walked. What would you be measuring? Circle the correct answer.

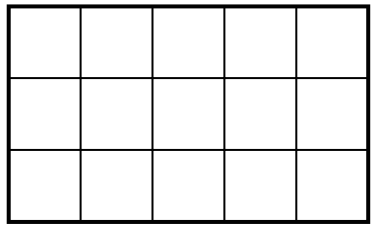
a. area

b. perimeter

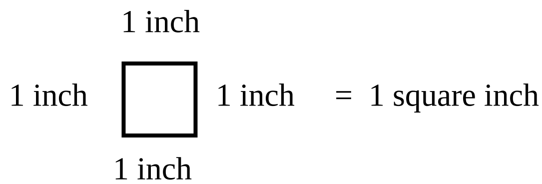
c. shape

d. strength

5. Use this rectangle to answer the following questions.



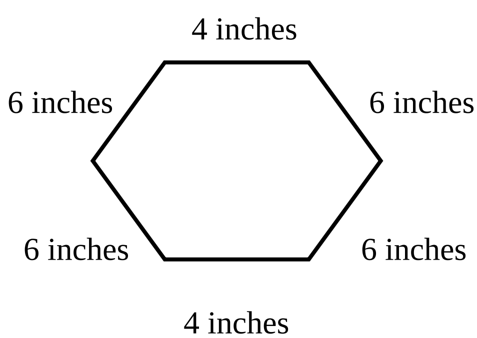
One small square in the large rectangle was equal to 1 square inch (each side was 1 inch).



a. What would be the total distance around the outside of the large rectangle? __________

b. What would be the total space the large rectangle takes up? ________________

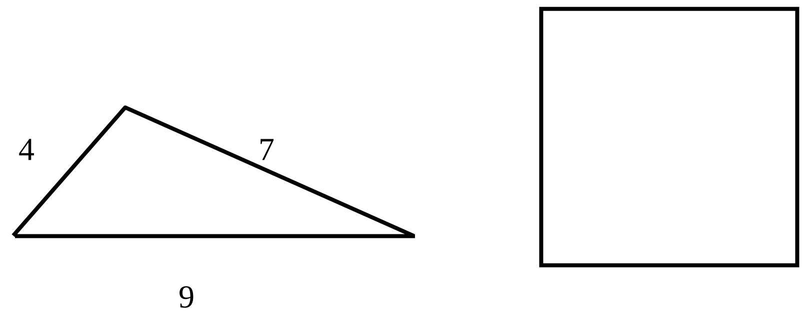
6. What is the perimeter of this hexagon?



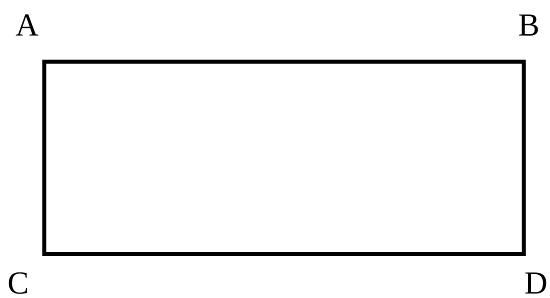
Perimeter = _________________

7. If both the square and the triangle below have the same perimeter, what is the length of each side of the square? Circle the correct response.

a. 4
b. 5
c. 6
d. 7



8. Describe a procedure (that is draw a proof) for showing that a closed 4-sided closed figure is a rectangle. Use the rectangle drawn below –show, describe and label**.**



Describe your "proof"

9. You are preparing a fish camp and deciding what size cutting board to cut salmon on. A cutting board is usually rectangular. You decide that your cutting board will have a perimeter of 60 inches. Draw a cutting board (below) that has a 60- inch perimeter. Label the length and width of the cutting board.

10. a. Draw a figure whose perimeter is 24 units.

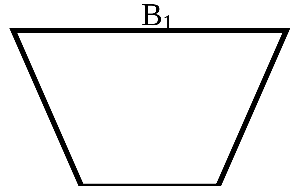
b. Make up a word problem in which you need to find the perimeter.

c. Can two different figures have the same area but different perimeters? Draw and explain your answer.

11. The formula for the area of a trapezoid is? Circle the correct answer.

a) $A = 1/2 (B_1 + B_2) H$
b) $A = 1/2 L \times W$
c) $A = 2 (B_1 + B_2)H$
d) $A = B_{1+} + B_2 + 2H$

B_1

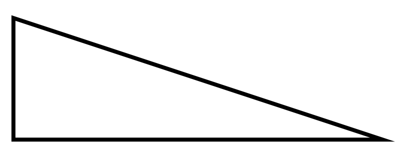


B_2

12. Find the area of the following triangle.

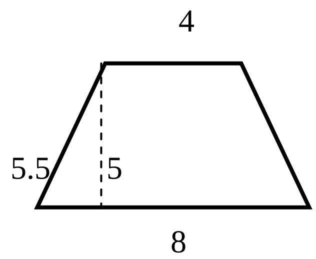
a. 32
b. 16
c. 24
d. 12

4



8

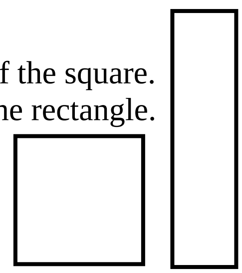
13. Find the area and the perimeter for the figure shown below.



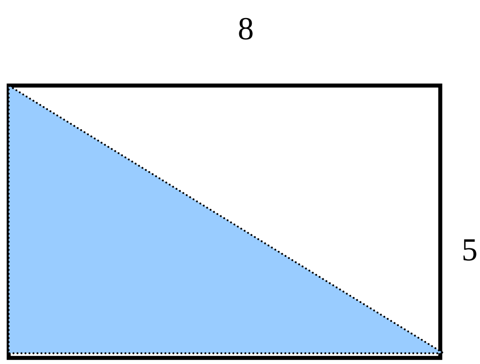
a. Area = ___________
b. Perimeter = _______

14 . The two figures below have the same area. The side of the square is twice as long as the width of the rectangle. Circle the rule or rules that are true.

a. The perimeter for the rectangle is greater than the perimeter of the square.
b. The perimeter for the square is greater than the perimeter of the rectangle.
c. The perimeter of A and B are the same.
d. Not enough information to solve this problem.



15. What is the PERIMETER of this rectangle? The area of the rectangle? The area of the shaded in part of the rectangle (the triangle)?

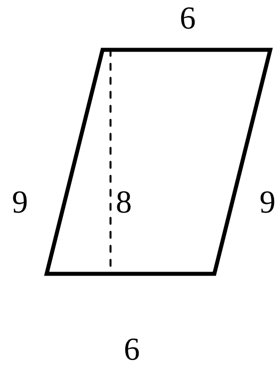


Circle the perimeter of the rectangle.

A) 13 meters
B) 26 meters
C) 40 meters
D) 80 meters
E) I don't know

Area of the rectangle = __________
Area of the triangle = ___________
16. Here is a parallelogram. Write a formula for finding the area of a parallelogram in the space

provided below. Write the formula below.



A.Write the formula here.

B. Draw and explain the formula (that is, how is the formula derived).

C. Calculate the area of this figure.

17. If you have two different regular shaped ice rinks each with a perimeter of 100 feet will the areas of each be different? Circle the correct answer.

a. All differently shaped ice rinks will have the same area since their perimeter equals 100 feet.
b. All differently shaped ice rinks will have different areas because area is not calculated from total perimeter
c. You can't tell from the information.

You want to build a rectangular skating rink with the maximum area but still has a perimeter of a 100 feet. Draw that "skating rink" below and label its length and width. Then explain why this rink has the maximum area. Draw below here.

Explain____
____
____

APPENDIX D

Development and Comparability of Pre- and Post-test Items

We developed pre- and post-test items in the following way. For comparative purposes and for well-developed test items, we gathered questions from the National Assessment of Educational Progress (NAEP) and the Third International Mathematics and Science Study (TIMSS). We generated additional test items that reflect the major mathematical subject components of the module around the concepts of shape, perimeter, and area. To ensure that test items adequately measured the students' initial knowledge of math and their improvement after three weeks of study, we piloted both pre- and post-test instruments with one sixth grade class, two seventh grade classes, and two eighth grade classes.

Specifically, the pilot tests assessed the comparability in difficulty between the pre- and post-test as well as differentiated the difficulty of the test items. Issues of readability, clarity, difficulty, range of types, and length of test were considered through the piloting. According to feedback we received from teachers and the pilot data, we adjusted the instrument; however, we did not perform any statistical analysis for comparability at that time. More specifically, teachers suggested the items test both simpler concepts about shape and more difficult concepts, such as the idea of perimeter held constant while dimensions and area change. In these ways, the test itself would be more sensitive to students' growing knowledge. Teachers also suggested that the test be shorter, which was met by taking out all of the ambiguous items originally used to assess students' creativity.

The post-test was specifically created at a slightly higher level of difficulty. Our reasoning followed that used by the "Solomon Four-Group Design" (Gliner & Morgan, 2000); its purpose is to mitigate the effects of the pre-test on the post-test. In other words, it can be argued that students can gain a small form of instruction from test situations. By merely taking the pre-test, there is an effect in the students’ understanding of the concepts being tested. Also, since the post-test was given three weeks later, students will have matured between testing times. Because of limitations of sample size we were not able to include a post-test only control group.

Once the tests were finalized the rubric was created. The pre-test had 14 questions, each worth five points, and three questions worth 10 points. The post-test had 13 questions, each worth five points, two questions worth 10 points, and two questions worth 7.5 points each. Partial credit was given for certain questions: (1) in the case of multiple correct solutions or (2) when students demonstrated partial understanding on open-ended questions. Thus the raw score is equivalent to a percentage score. The percentage score is used throughout this paper.

Table D1 shows a method of recoding the items in both the pre- and post-tests based on item description. This allows us to match the items between the tests. Note that there are five items on the pre-test that do not match items on the post-test and similarly four unique items on the post-test. Note also that there are two pairs of items on the post-test overlapping items on the pre-test.

For the questions that match, it is simple to argue that the questions on the post-test are comparable to those on the pre-test at a sometimes slightly more difficult level to account for the maturation of students and the testing circumstances. For example, consider the first question on both the pre- and the post-test. Given four shapes, the student must identify the square on the pre-test but the parallelogram on the post-test. (See Appendices B and C for the tests in full).

Table D1: Recoding of Pre- and Post-Test Items by Item Description

Pre-test Item Label	Post-test Item Label	Recoded Label	Description of Item
1	1	*a*	identify shape
2	2	*b*	properties of rectangle
3		*c*	quadrilateral properties
4	3	*d*	identify quadrilateral
5	6	*e*	perimeter formula
6	4	*f*	basic definition of area/perimeter - inside or outside
7		*g*	2 different rectangles, same perimeter
8	7	*h*	perimeter vs. dimensions
9		*i*	comparing area of different shapes - open-ended
10	5b	*j*	calculate area of rectangle
11	5a	*k*	calculate perimeter of rectangle
12a	9, 10a	*l*	draw figure with specified area or perimeter
12b	10b	*m*	create a word problem to match 12a
12c	10c	*n*	2 rectangles, same perimeter, different areas or vice versa
13a	14	*o*	comparing perimeter or area of 2 rectangles
13b	15	*p*	calculating perimeter of rectangles
14	11	*q*	identify formula - area of parallelogram or trapezoid
15		*r*	vocabulary - circumference is same as perimeter
16a	13b	*s*	calculate perimeter parallelogram or trapezoid
16b	13a, 16c	*t*	calculate area of parallelogram or trapezoid
17a		*u*	calculate perimeter of triangle
17b	12	*v*	calculate area of triangle
	8	*w*	proof of a rectangle
	16a	*x*	create formula for area of parallelogram
	16b	*y*	proof of formula for 16a
	17	*z*	maximize area of rectangle for fixed perimeter

As for the questions that do not qualify as similar between pre- and post-tests, we can argue that those on the post-test are more difficult than those extras on the pre-test but not to the extent of lack of comparability between tests. For example, the proof type problems (*w* and *y*) are obviously addressing students' understanding at a deeper level than the vocabulary or perimeter calculations in *r* and *u*. However, item *g* on the pre-test asks students to draw two different rectangles with the same perimeter, while item *z* on the post-test assesses students' deeper understanding on this topic by asking for the rectangle (having the same perimeter) with maximum area.

In conclusion, we believe the pre-test and post-test were comparable to each other for the purpose of measuring students' growth on the subject matter of shape, perimeter, and area. In the future, more concern will be given to address the issue of comparability of test items by using procedures such as the Pearson's coefficient.